Efficient Predictive Steering of Local Clocks in GPS-based Timekeeping

Yuriy S. Shmaliy, Oscar Ibarra-Manzano, and Luis Arceo-Miquel
FIMEE, Guanajuato University, Salamanca, Gto, 36885, Mexico
Email: shmaliy@salamanca.ugto.mx

***Abstract*— We discuss optimal synchronization of local clocks by the Global Positioning System (GPS) one pulse per second (1PPS) timing signals. To eliminate sawtooth errors peculiar to the 1PPS signals and optimally steer the clock errors each M seconds, we propose using a ramp predictive finite impulse response (FIR) filter that is known to be optimal for clock models on large averaging horizons. A low-pass filter is used to smooth the hold filter output between the optimally predicted points. A GPS locked crystal clock has been investigated in detail in terms of the time interval error, Allan deviation, and precision time protocol (PTP) variance. A high efficiency implementation of the proposed synchronization algorithm is demonstrated experimentally.**

I. Introduction

The need to synchronize local time scales arises with different allowed uncertainties in digital communications [1] [2], bistatic radars [3], telephone networks [4], networked measurement and control systems [5], space systems [6] [7], computer nets [8], etc. To discipline the clocks, commercially available Global Positioning System (GPS) timing receivers are often used conveying the reference time to the locked clock loop via the one pulse per second (1PPS) output. An organization of the loop is provided such that the clock time error ranges over time below an allowed threshold that, for digital communication networks, is specified in [9].

GPS disciplining of local clocks can be organized in two ways:

- Both the time error and the frequency offset are corrected via the GPS-disciplined oscillator (GPSDO).
- Only the time error is adjusted in controlled digital parts of the clocks without touching a local oscillator.

The basic block diagram of a GPS-disciplined clock is shown in Fig. 1. The 1PPS output of the GPS timing receiver is used as a reference, in which the regular time delays caused by signal propagation and other factors are eliminated at the early stage. The time difference between the 1PPS that is accurate but not precise, owing to noise, and the 1s output of a local clock that is precise but inherently not accurate, is measured each second with a high resolution time interval error (TIE) counter. The clock TIE is then estimated by the filter to produce a synchronizing signal intended to discipline the clock time scale.

In a locked clock time scale, noise mostly depends on precision of a local oscillator and time error departures are limited by the accuracy of the reference time signals. The high accuracy of synchronization is achieved using the filter, the design of which can vary. Averaging and low-order low-pass (LP) filters are typical for commercially manufactured GPSDOs. It has been proposed in [10] to use averaging and integrating filters along with the phase locked loop (PLL). In [11], the use of the 3-order PLL is proposed in order to minimize phase errors. A linear least squares estimator is exploited in [12] to discipline a rubidium standard. Some authors propose applying the Kalman filters [13]–[15] and even neural networks [16]. Experimental investigations of several GPSDOs utilizing different filters can be found in [17] [18].

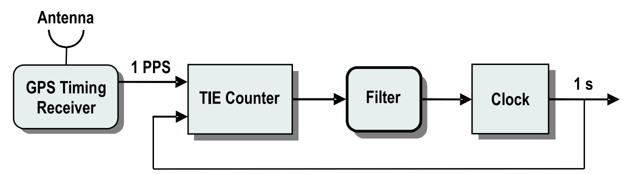

Fig. 1. A generalized structure of GPS disciplining of local clocks by the 1PPS timing signals.

In this paper, we propose a novel approach for optimal GPS synchronization of local time scales using a predictive ramp finite impulse response (FIR) filter. The latter is known to be optimal for clock models [19], by large averaging horizons typically used in timekeeping. In our loop (Fig. 1) we use the Timing SynPaQ III GPS Sensor (Synergy Systems, LLC, San Diego, CA). The Frequency Counter SR620 (Stanford Research System, Inc., Sunnyvale, CA) is used as the TIE counter. We exploit the oven controlled crystal oscillator (OCXO) imbedded in the SR620 to form the local time scale with a high resolution divider. For simultaneous measurement of actual time errors, the reference Cesium Frequency Standard CsIII (Symmetricom, Inc., San Jose, CA) is used.

II. The Synchronization Loop Model

With respect to the clock first state x_n representing the TIE, the synchronization loop (Fig. 1) can be modeled as shown in Fig. 2. Here, the reference time uncertainty v_n (Fig. 3a) is caused by different satellites in a view (GPS time errors), measurement errors, and other factors [20]. The uniformly distributed sawtooth noise w_n (Fig. 3b) [21] is induced by the receiver to range from -50 ns to 50 ns owing to the principle of the 1PPS signal formation [22]. In measurements with sawtooth, the reference signal noise s_n is composed by

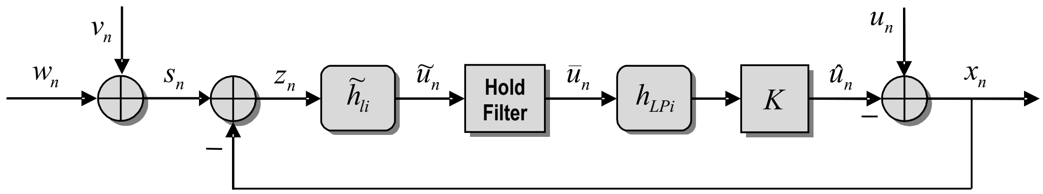

Fig. 2. The loop model of GPS synchronization of a local clock, by the 1PPS timing signals.

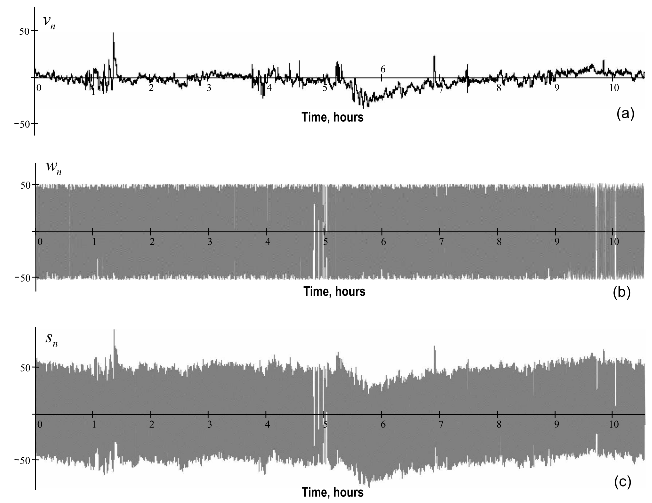

Fig. 3. Typical time errors of a Motorola-based GPS timing receiver: (a) time uncertainty v_n caused by different satellites in a view and other random factors, (b) sawtooth noise w_n induced by the receiver, and (c) time error s_n in the 1PPS reference signal.

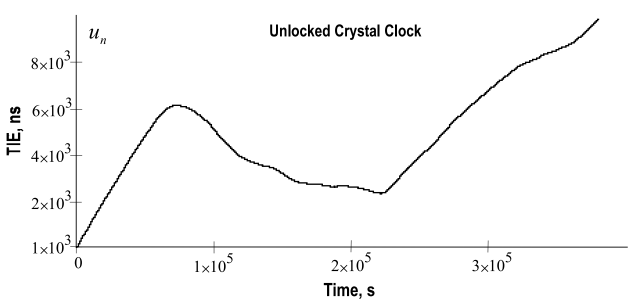

Fig. 4. Typical behavior of the TIE of an unlocked crystal clock.

an additive sum of v_n and w_n (Fig. 3c). In some GPS timing receivers, the negative sawtooth correction code is available in the protocol. If the sawtooth correction is applied, then one can approximately let $s_n = v_n$. A typical behavior of the clock time error u_n, caused by a local OCXO, is shown in Fig. 4.

The loop operates as follows. The value x_n is subtracted from s_n by the TIE counter to measure $z_n = s_n - x_n$. It is implied that N such measurements are provided in the nearest past sample history from $n-N$ to $n-1$. To predict u_n at a current point n, the predictive FIR filter with the gain $\tilde{h}_{li}$ of degree l is used. The predicted value $\tilde{u}_n$ is held as $\bar{u}_n$ by a hold filter. Predictions are smoothed by an LP filter with the gain $h_{_{LPi}}$ and gain K applied to be $\hat{u}_n$. Finally, $\hat{u}_n$ is subtracted from u_n to yield $x_n = u_n - \hat{u}_n$. Compensated u_n, the clock TIE x_n, in an ideal case, reaches zero.

A. Predictive FIR Filter

It has been shown in [23] and [24] that the unbiased FIR filters produce lower errors than the Kalman filter for GPS-based measurements of the TIE. Contrary to the recursive Kalman filters, such filters are inherently bounded input/bounded output stable and have better robustness against uncertainty v_n and round-off errors [25]. On the other hand, Lepek showed in [26] that linear predictors are optimal or close to optimal for the prediction of clock instabilities. We therefore infer that the unbiased 1-degree ramp FIR filter

$$h_{1i} = \begin{cases} \frac{2(2N-1)-6i}{N(N+1)} & , \quad 0 \leqslant i \leqslant N-1 \\ 0 & , \quad \text{otherwise} \end{cases} \tag{1}$$

derived in [27] and becoming optimal [19] in the sense of the minimum mean square error (MSE), by large $N \gg 1$, can be used in the loop (Fig. 2) after modified to be 1-step predictive. Such a modification has been provided in [28], [29] for p steps. Setting $p = 1$, we arrive at the gain of the 1-step predictive ramp FIR filter, (60) in [28] and (31) in [29],

$$\tilde{h}_{1i} = \begin{cases} \frac{2(2N+1)-6i}{N(N-1)} & , \quad 1 \leqslant i \leqslant N \\ 0 & , \quad \text{otherwise} \end{cases} , \tag{2}$$

originally derived by Heinonen and Neuvo in [30]. Contrary to (1), this filter has low efficiency [29] with small N, because of an uncertainty at $N = 1$. However, this does not mean too much for clock applications claiming $N \gg 1$.

By (2), the 1-step predictive estimate can be found as

$$\tilde{u}_n = -\sum_{i=1}^{N} \tilde{h}_{1i} z_{n-i} , \tag{3}$$

meaning that $\tilde{u}_n$ is produced by the discrete convolution via the past values of z_n taken from $n-1$ to $n-N$.

B. Hold Filter

To hold $\tilde{u}_n$ over time for continued steering of x_n between the optimally defined values, a hold filter is used such that

$$\bar{u}_n = \tilde{u}_{\lfloor \frac{n}{M} \rfloor M} , \tag{4}$$

where $\lfloor \frac{n}{M} \rfloor$ is an integer part of n/M. By (4), the input and output values of the hold filter become equal when n is multiple to M. In the gap between two such neighboring points, the output value of the hold filter is constant.

C. LP Filter and Gain K

For multiple steering of clock errors with period τM, where τ is a sampling time and M is a number of sampling times, the hold filter produces a step signal $\bar{u}_n$. Applied straightforwardly to the clock, $\bar{u}_n$ obtains optimal steering and assures that x_n ranges within minimum bounds around zero. On the other hand, an impulsive $\bar{u}_n$ may not be appropriate to obtain the required noise performance of a locked clock.

To smooth $\bar{u}_n$, an LP filter with the impulse response $h_{_{LPi}}$ is included. Contrary to $\tilde{h}_{1i}$ producing optimal estimates, $h_{_{LPi}}$ is intended to smooth the step signal $\bar{u}_n$ without attenuation. We shall show below experimentally that the 1-order LP filter with

$$h_{_{LPi}} = \begin{cases} Ae^{-\frac{\tau}{T}i} & , \quad i \geqslant 0 \\ 0 & , \quad i < 0 \end{cases} , \tag{5}$$

where T is a time constant and $A = 1 - e^{-\tau/T}$, is able to fit the demands for precise synchronization.

Finally, if the LP filter does not attenuate $\bar{u}_n$, one may set $K = 1$.

III. Synchronization Algorithm

The GPS synchronization algorithm is illustrated in Fig. 5. It is implied that measurements of z_n are provided with high resolution and the clock is designed such that $\hat{u}_n$ steers its errors directly and linearly.

The FIR filter processes data from 0 to $N-1$ producing a prediction $\tilde{u}_n$ at N (Fig. 5a). This value is held by the hold filter (4) starting at $n = N$ (Fig. 5c) and steers x_n (Fig. 5b). Accordingly, x_n jumps at $n = N$ closely to zero and thereafter behaves as in Fig. 5d (bold) starting at $n = N$. The function $\bar{u}_n$ is then smoothed (dashed curve) by an LP filter.

For multiple synchronization with period M, a smoothed value $\hat{u}_n$ can be found, using the discrete convolution and (2)–(5), as

$$\hat{u}_n = K \sum_{l=0}^{L-1} h_{_{LPl}} \tilde{u}_{\lfloor \frac{n-l}{M} \rfloor M} , \tag{6}$$

where L is a reasonable length of the impulse response $h_{_{LPi}}$. Substituting (6) to $x_n = u_n - \hat{u}_n$ and accounting for (3) and $z_n = s_n - x_n$ allows writing the loop general equation as

$$\begin{aligned} x_n \quad = \quad & u_n - K \sum_{l=0}^{L-1} \sum_{i=1}^{N} h_{_{LPl}} \tilde{h}_{1i} x_{\lfloor \frac{n-l}{M} \rfloor M-i} \\ & + K \sum_{l=0}^{L-1} \sum_{i=1}^{N} h_{_{LPl}} \tilde{h}_{1i} s_{\lfloor \frac{n-l}{M} \rfloor M-i} \end{aligned} \tag{7}$$

where the difference between u_n and the first block of sums represents the regular error (bias) and the remaining term the noise in x_n.

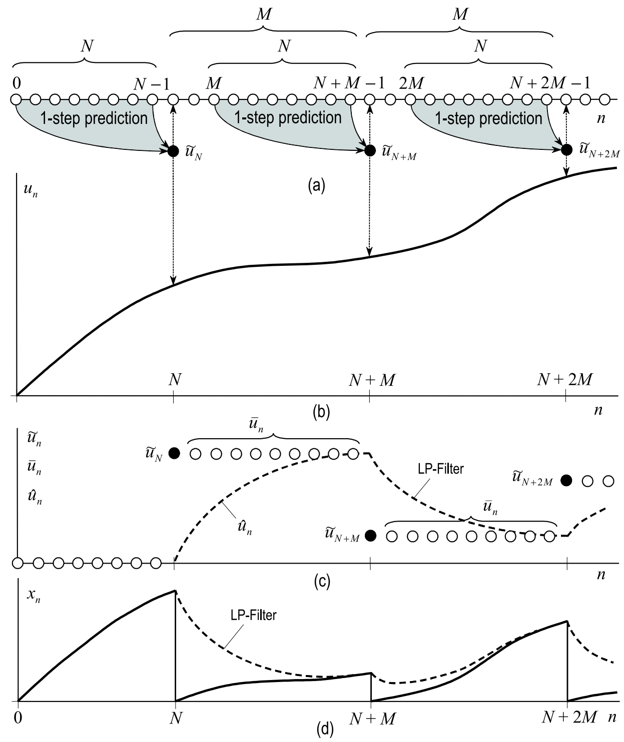

Fig. 5. The GPS synchronization algorithm for $M > N$: (a) time diagram, (b) time error u_n of an unlocked clock, (c) optimal predictive estimate $\tilde{u}_n$, hold filter output $\bar{u}_n$, and smoothed estimate $\hat{u}_n$, and (d) time error x_n of a locked clock: piecewise (bold) and smoothed by an LP filter (dashed).

IV. Experimental Investigations

In this Section, the loop (Fig. 2) modeled by (7) in accordance with the algorithm (Fig. 5) is examined in terms of the TIE, Allan deviation, and PTP variance. Following [31], the optimum value of τ is allowed to be $\tau_{\text{opt}} = 1$ s. We let $M = N$, $K = 1$, and determine both N_{opt} and T_{opt} experimentally in the sense of the minimum synchronization MSE for the crystal clock TIE measured with and without the sawtooth.

A. Time Interval Error

In the first experiment, we investigate the TIE x_n of a clock locked with and without the sawtooth correction.

For measurements with sawtooth errors, $s_n = v_n + w_n$, the optimum N has been experimentally found, following [31], to be $N_{\text{opt}} \cong 250$. By this value and without smoothing, $T = 0$, the TIE behaves as in Fig. 6a. By $T_{\text{opt}} \cong 2000$, the LP filter minimizes the MSE in the TIE as shown in Fig. 6b.

For sawtooth-less measurements[1], $s_n = v_n$, the optimal N has appeared to be $N_{\text{opt}} \cong 150$, leading to the results reflected in Fig. 7a and Fig. 7b. One infers that sawtooth correction does not change the picture cardinally, although it slightly draws together the TIE x_n bounds.

[1]The term "sawtooth-less measurement" is used in the sense of the GPS-based sawtooth measurement corrected with the prediction negative sawtooth code provided by the receiver protocol.

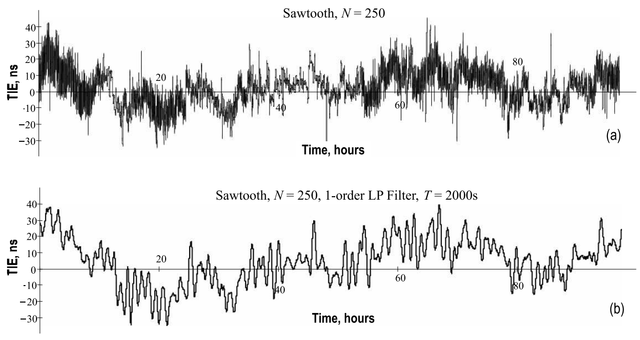

Fig. 6. Time error x_n of the optimally GPS locked crystal clock with sawtooth measurements, $s_n = v_n + w_n$: (a) piecewise steering with $N = 250$ and (b) smoothed steering with the 1-order LP filter, $T = 2000$s.

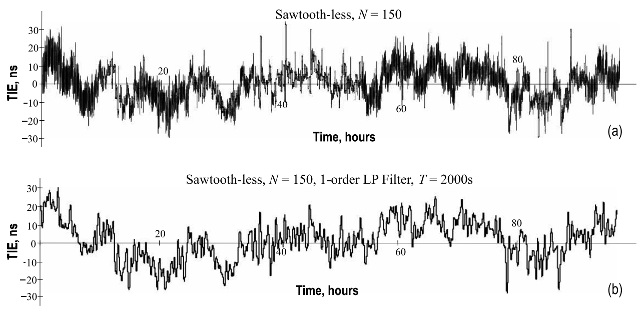

Fig. 7. Time error x_n of the optimally GPS locked crystal clock with sawtooth-less measurements, $s_n = v_n$: (a) piecewise steering with $N = 150$ and (b) smoothed steering with the 1-order LP filter, $T = 2000$s.

We finally assume the uncertainty-less steering, $s_n = w_n$, find $N_{\text{opt}} \cong 250$, and arrive at the results exhibited in Fig. 8. An instant conclusion is that, by $v_n = 0$, the TIE lies very closely to zero at almost each of the points $n = N + mM$, in contrast to both Fig. 6a and Fig. 7a. That, however, does not lead to substantial narrowing the TIE range. The latter still fundamentally depends on the time error behavior of the unlocked clock (Fig. 4), irrespective of noise in the reference signal. A comparison of Fig. 6b, Fig. 7b, and Fig. 8b assures us in this fact.

B. Allan Deviation

Setting the experimentally determined values of $N_{\text{opt}} = 250$ and $N_{\text{opt}} = 150$ for sawtooth and sawtooth-less measurements, respectively, we now investigate the Allan deviation of a locked clock for different T of the LP filter.

Inherently, the Allan deviation of the 1PPS with sawtooth, $s_n = v_n + w_n$ (Fig. 9a), traces upper than that without sawtooth, $s_n = v_n$ (Fig. 9b). By the optimal FIR filter, the former is lowered substantially (Fig. 9a), whereas the latter remains almost unaltered (Fig. 9b). In both cases, an LP filter forms the final pictures shown in Fig. 9. Namely, with small averaging times, the Allan deviation is determined by the unlocked clock and with large averaging times by the optimal FIR filter (or sawtooth-less measurements). Also,

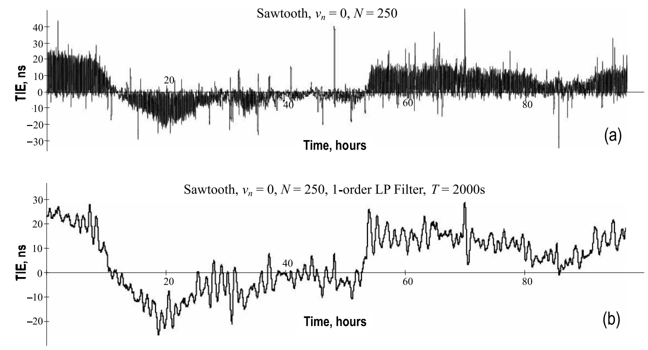

Fig. 8. Limiting time error x_n of the optimally GPS locked crystal clock with measurements without uncertainty, $s_n = w_n$: (a) piecewise steering with $N = 250$ and (b) smoothed steering with the 1-order LP filter, $T = 2000$s.

the Allan deviation exhibits an excursion, the peak value of which reaches a minimum when $T_{\text{opt}} \cong 2000$ s. This local nonuniformity displaces to the right, by $T > T_{\text{opt}}$, and to the left, by $T < T_{\text{opt}}$.

C. Precision Time Protocol Variance

Because the PTP variance[2] is stated in [5] to be the main measure of errors in locked clocks, the relevant investigations were provided. The results are illustrated in Fig. 10. Here, we selected $T \cong 1000$ s to be optimum, once it provides better placement of the locked clock PTP deviation (square root of the PTP variance) below the TDEV mask specified by [9] for digital communication networks. This value can differ for another masks.

V. An Analysis

Some generalizations can now be provided regarding the proposed optimal GPS synchronization algorithm and performance of a locked clock.

1) Effect of the optimal FIR filter: It follows, by comparing "GPS 1PPS, v_n" and "FIR Filter" in Fig. 9b and Fig. 10b, that the optimal FIR filter eliminates the sawtooth error w_n similarly to sawtooth correction and does not substantially affect the uncertainty v_n. That means that the proposed loop would operate with almost equal efficiency for commercially available GPS timing receivers with and without sawtooth correction.

2) Optimum parameters: The minimum MSE in x_n can be reached with the following optimized parameters:

- *Sampling time*: $\tau_{\text{opt}} = 1$ s [31].
- *Averaging horizon*: From Fig. 9a, it follows that N_{opt} lies about the cross point of the Allan deviation parts with the slopes $\tau^{-1/2}$ and τ^0.
- *Period of synchronization*: Although M can be arbitrary, the filter horizon, by $M < N$, overlaps both free and steered points (Fig. 5), causing uncertainty errors. On the

[2]For $M = N$, a square root of the PTP variance $\sigma^2_{PTP}(\bar\tau)$, where $\bar\tau$ is the averaging time, is equal to the time deviation (TDEV) specified in [9]. The PTP variance relates to the Allan variance $\sigma^2_y(\bar\tau)$ as $\sigma^2_{PTP} = \bar\tau^2\sigma^2_y/3$.

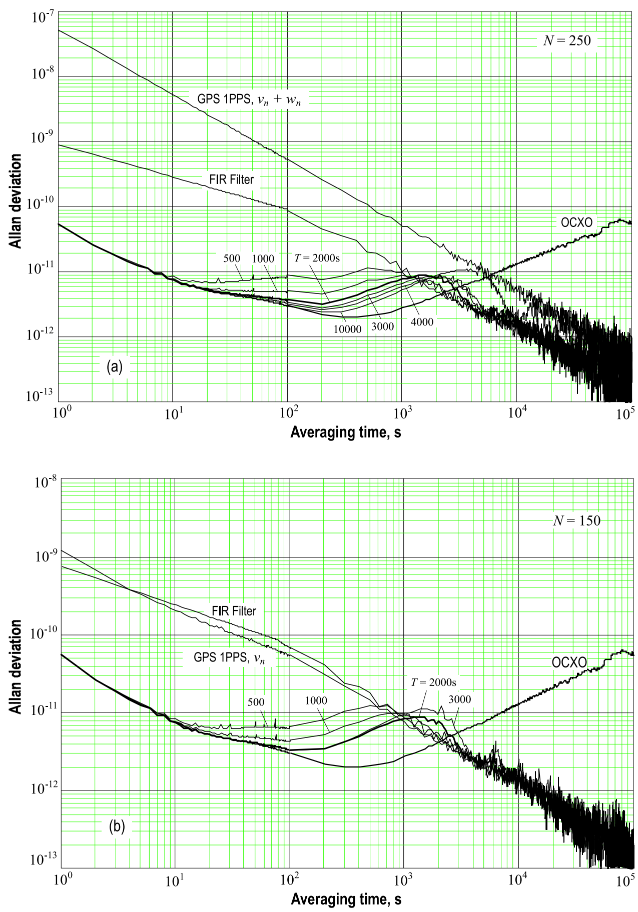

Fig. 9. Allan deviation of the GPS locked crystal clock for different T,s of the 1-order LP filter: (a) sawtooth measurement $s_n = v_n + w_n$ with $N_{\rm opt} = 250$ and (b) sawtooth-less measurement $s_n = v_n$ with $N_{\rm opt} = 150$.

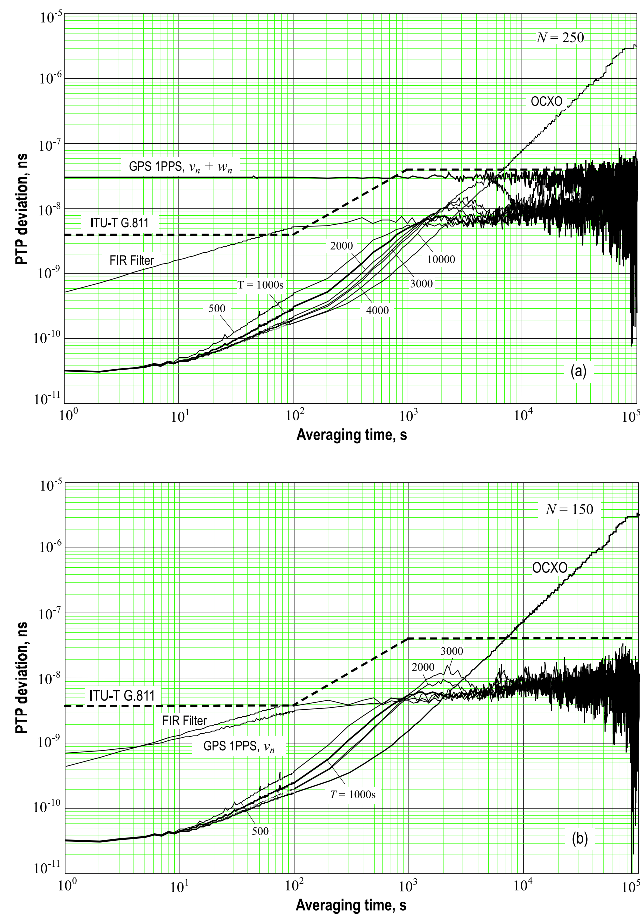

Fig. 10. PTP deviation of the GPS locked crystal clock for different T,s of the 1-order LP filter: (a) sawtooth measurement $s_n = v_n + w_n$ with $N_{\rm opt} = 250$ and (b) sawtooth-less measurement $s_n = v_n$ with $N_{\rm opt} = 150$.

other hand, by $M > N$, synchronization is inefficient, because the points from $n = N$ to $n = M - N$ are not processed. Thus, $M_{\rm opt} = N_{\rm opt}$.

- *Time constant of the LP filter*: It follows from Fig. 9a that $T_{\rm opt}$ lies about the cross point between "FIR Filter" and "OCXO."

Summarizing, Fig. 11 sketches a generalized picture for the Allan deviation of a locked crystal clock. To obtain the "bold" curve, a designer must first determine $N_{\rm opt}$ about the cross point between $\tau^{-1/2}$ and τ^0. By $N_{\rm opt}$, the function "A" is lowered to "B" corresponding to sawtooth-less measurements. Then $T_{\rm opt}$ is specified about the cross point between "B" and "C" and the goal is reached. Let us notice that an excursion in the Allan deviation cannot be avoided from the standpoint of control. It can only be minimized and its position optimized by $T_{\rm opt}$.

VI. Concluding Remarks

In this paper we proposed and examined a novel optimal synchronization loop, in which a local clock is disciplined by the GPS timing 1PPS signals using the optimal predictive ramp FIR filter and a smoothing 1-order LP filter. Some generalizations of the studies can be found in Section V. The main overall conclusion is that the optimal FIR filter eliminates the sawtooth similarly to sawtooth correction and does not affect substantially the uncertainty v_n. The proposed loop can therefore operate with almost equal efficiency for different kinds of commercially available GPS timing receivers, which 1PPS signals are formed with and without the sawtooth. We finally notice that, in out experiments, high-order smoothing LP filters did not contribute with lower errors to the loop. Even so, possible optimization of the LP filter is currently under investigation.

References

[1] S. M. Bass, P. Vlitas, and M. Garvey, "A redundant timing source for digital communication network synchronization," in *Proc. 44th Annu. Symp. on Freq. Contr.,* pp. 117–126, 1990.

[2] Q. D. Hua, M. S. Williams, J. Evans, and M. A. Trippy, "Use of the NAVSTAR Global Positning System (GPS) in the defence communications system (DCS)," in *Proc. 1993 IEEE Int. Freq. Cont. Symp.*, pp. 40–44, 1993.

[3] N. J. Willis, *Bistatic Radars*, SciTech Publ., 2005.

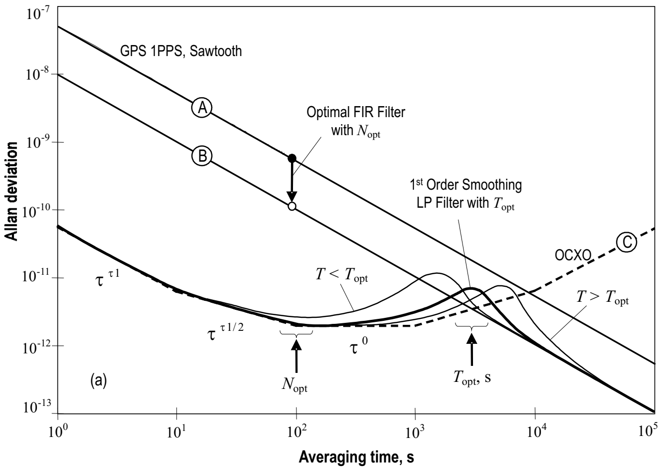

Fig. 11. The Allan deviation of a local crystal clock synchronized by GPS 1PPS timing signals.

[4] J. W. Pan, "Present and future of synchronization in the US telephone network," in *IEEE Trans. on Ultrason., Ferroel. and Freq. Contr.,* vol. 34, no. 6, pp. 629–638, Nov. 1987.

[5] IEEE Standard 1588^{TM}-2002 for a precision clock synchronization protocol for networked measurement and control systems.

[6] T. Iwata, M. Imae, T. Suzuyama, H. Murakami, Y. Kawasaki, "Simulation and ground experiments of remote synchronization system for on-board crystal oscillator of Quazi-Zenith Satelite System," in *J. of the Institute of Navigation,* vol. 53, no. 4, pp. 231–235, 2006.

[7] L. J. Rueger, J. R. Norton, and P. T. Lasewicz, "Long-term performance of precision crystal oscillators in a near-Earth orbital environment," in *IEEE Trans. on Ultrason., Ferroel. and Freq. Contr.,* vol. 40, no. 5, pp. 528–531, Sep. 1993.

[8] J. Levine, "Time synchronization over the Internet using an adaptive frequency-locked loop," in *IEEE Trans. on Ultrason., Ferroel. and Freq. Contr.,* vol. 46, no. 4, pp. 888–896, Jul. 1999.

[9] ITU-T Recommendation G.811: Timing characteristics of primary reference clocks, 1997.

[10] N. C. Helsby, "GPS disciplined offset-frequency quartz oscillator," in *Proc. IEEE Int. Freq. Contr. Symp.,*, pp. 435–439, 2003.

[11] B. M. Penrod, "Adaptive temperature compensation of GPS disciplined quart and rubidium oscillators," in *Proc. IEEE Int. Freq. Contr. Symp.,*, pp. 980–987, 1996.

[12] A. MacIntyre and S. R. Stein, "A disciplined rubidium oscillator," in *Proc. 40th Ann. Freq. Contr. Symp.,*, pp. 465–469, 1986.

[13] S. R. Stein and R. L. Filler, "Kalman filter analysis for real time applications of clocks and oscillators," in *Proc. 42th Annu. Freq. Contr. Symp.,* pp. 447–452, 1988.

[14] Yu. S. Shmaliy, A. V. Marienko, and A. V. Savchuk, "GPS-based optimal Kalman estimation of time error, frequency offset, and aging," in *Proc. 31st Ann. PTTI Mtg.,* pp. 431–440, 1999.

[15] C. Greenhall, "Kalman plus weights: a time scale algorithm," in *Proc. 33rd Annu. PTTI Mtg,*, pp. 445–454, 2001.

[16] S. Y. Chen, C. Y. Yao, G. Xiao, Y. S. Ying, and W. L. Wang, "Fault detection and prediction of clocks and timers based on computer audition and probabilistic neural networks," *Lecture Notes in Computer Science*, vol. 3512, pp. 952–959, 2005.

[17] J. A. Davis and J. M. Furlong, "A study examining the possibility of obtaining traceability to UK National Standards of time and frequency using GPS-disciplined oscillators," in *Proc. 29th Ann. Precise Time and Time Interval (PTTI) Mtg.,*, pp. 329–343, 1997.

[18] M. A. Lombardi, A. N. Novak, and V. S. Zhang, "Characterizing the performance of GPS disciplined oscillators with respect to UTC(NIST)," in *Proc. IEEE Int. Freq. Contr. Symp.,*, pp. 677–684, 2005.

[19] Yu. S. Shmaliy, "On real time optimal FIR estimation of linear TIE models of local clocks," in *IEEE Trans. on Ultrason., Ferroel. and Freq. Contr.,* vol. 54, no. 11, pp. 2403–2406, May 2006.

[20] P. Eskelinen, "Problems in estimating some timing uncertainties of commercial frequency and time standards," *IEEE Trans. Instrum. Measur.*, vol. 48, no. 1, pp. 62–65, Feb. 1999.

[21] Yu. S. Shmaliy, O. Ibarra-Manzano, L. Arceo-Miquel, and J. Muñoz-Diaz, "An analysis of sawtooth noise in the timing SynPaQ III GPS sensor," *Sensors & Transducers J.*, vol. 79, no. 5, pp. 1151–1156, May 2007.

[22] R. M. Hambly and T. A. Clark, "Critical evaluation of the Motorola M12+ GPS timing receiver vs. the master clock at the United States Naval Observatory, Washington, DC," in *Proc. 34th Annu. PTTI Mtg,* pp. 109–115, 2002.

[23] Yu. S. Shmaliy, "An unbiased FIR filter for TIE model of a local clock in applications to GPS-based timekeeping," in *IEEE Trans. on Ultrason., Ferroel. and Freq. Contr.,* vol. 53, no. 5, pp. 862–870, May 2006.

[24] Yu. S. Shmaliy, O. Ibarra-Manzano, L. Arceo-Miquel, and J. Muñoz-Diaz, "A thinning algorithm for GPS-based unbiased FIR estimation of a clock TIE model," in *Measurement,* vol. 41, no. 5, pp. 538–550, Jun. 2008.

[25] W. H. Kwon and S. Han, *Receding Horizon Control: Model Predictive Control for State Models*, Springer, London, 2005.

[26] A Lepek, "Clock prediction and characterization," *Metrologia*, vol. 34, no. 5, pp. 379-386, Oct. 1997.

[27] Yu. S. Shmaliy, "A simple optimally unbiased MA filter for timekeeping," in *IEEE Trans. on Ultrason., Ferroel. and Freq. Contr.,* vol. 49, no. 6, pp. 789-797, Jun. 2002.

[28] Yu. S. Shmaliy, "An unbiased p-step predictive FIR filter for a class of noise-free discrete-time models with independently observed states," in *Signal, Image and Video Process.,* vol.3, no.2, pp. 127-135, Jun. 2009.

[29] Yu. S. Shmaliy and L. Arceo-Miquel, "Efficient predictive estimator for holdover in GPS-based clock synchronization," in *IEEE Trans. on Ultrason., Ferroel. and Freq. Contr.,* vol.55, no.10, pp. 2131-2139, Oct. 2008.

[30] P. Heinonen and Y. Neuvo, "FIR-median hybrid filters with predictive FIR substructures," *IEEE Trans. Acoust. Speech Signal Process.*, vol. 36, no. 6, pp. 892-899, Jun. 1988.

[31] Yu. S. Shmaliy, J. Muñoz-Diaz, and L. Arceo-Miquel, and "Optimal horizons for a one-parameter family of unbiased FIR filters," in *Digital Signal Process.,* vol. 18, no. 5, pp. 739–750, Sep. 2008.